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Development of analytical approach for linear switched reluctance motor and its validation by two dimensional FEA

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Abstract – This paper gives an overview of modelling technique to describe the nonlinear behaviour of saturated Linear Switched Reluctance Machine (LSRM). The approach is based on the inductance versus position and current. Taking into account the non-linearity of the magnetic circuit, models are expressed by either Fourier series or polynomials where the only first three components are considered. The results of these analytical approaches are compared with those obtained using finite element methods (FEM).

Keywords – linear switched reluctance motor, analytical model, flux linkage, magnetic saturation.

1. Introduction

The Linear Switched Reluctance Motor (LSRM) is always operated in the magnetically saturated mode to maximize the energy transfer. The magnetic flux linked by a single phase must be known to develop a sophisticated controller. The inherent magnetic nonlinearity of the LSRM must be taken into account by appropriate modelling of the machine characteristics, [1-6-8].

In a LSRM, the phase inductances and flux linkages vary with rotor position due to stator and rotor saliencies. The phase inductances and flux linkages at any rotor position also vary with the instantaneous phase currents because of magnetic saturation. However, these variations can be modelled analytically using the data obtained through FEM or through experiments. These analytical expressions are used to represent the LSRM dynamics and hence the machine performance can be obtained, [11-23-24-26-27-28-29].

Obviously, the high degree of nonlinearity makes very difficult the modelling of the flux linkage or the phase inductance. Many researchers have addressed the problem of calculating the inductance or flux linkage from rotor position and phase current analytically with various degrees of accuracy, [10-12-13-14-16].

In order to determine a refined model which describes the behavior of a saturated reluctant structure, there are basically two ways to represent the static LSRM characteristics. The first way is to plot the phase flux linkage variations with rotor position and phase current. The second way is to plot the phase inductance variation with rotor position at different phase currents. These static characteristics are highly nonlinear. Figure 1 shows a classification of the different LSRM modelling techniques, [2-3-7-9-19-20-35-39-40].

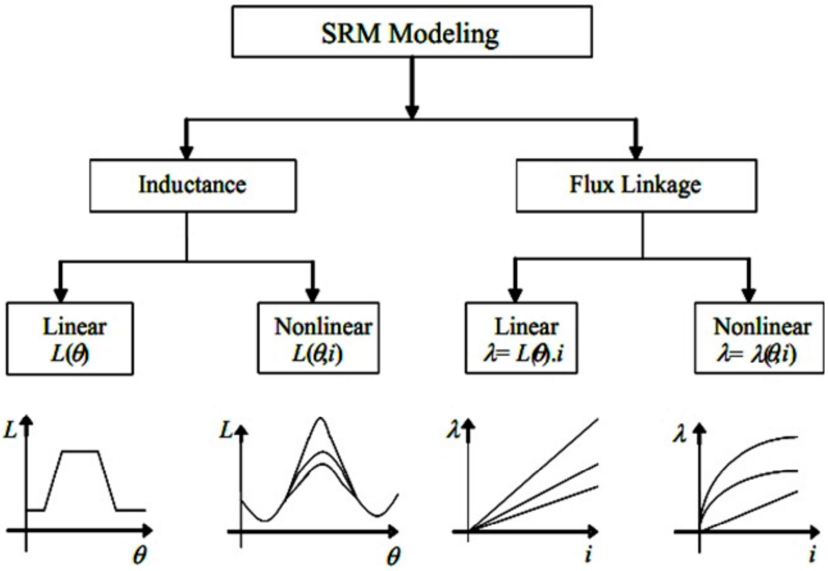


Fig.1. Classification of LSRM modelling.

Specifications of the designed prototype of the LSRM are shown in table 1.

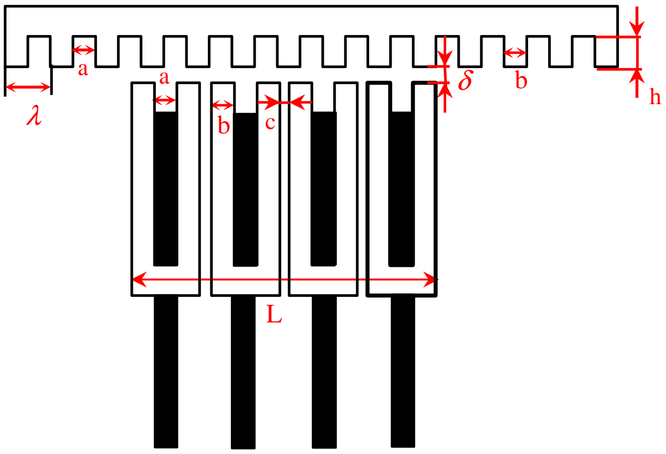


Fig. 2. Main dimensions of the conceived actuator.

Table 1. Motor mechanical and electrical parameters

Number of modules	4
Tooth width (b)	3mm
Slot width (a)	3mm
Tooth pitch (λ)	6mm
Phase separation (c)	1.5mm
Mover length	135 mm
stator length (L)	40.5 mm
Air gap width (δ)	0.1mm
Step size	1.5 mm
Number of turns per phase	520

2. Inductance -based model of LSRM

In a LSR machine, the reluctance of the magnetic path in a given phase changes with rotor movement. The reluctance is maximum when the stator and rotor poles are unaligned and minimum when the poles are aligned. This variation in reluctance reflects in the self-inductance of the respective stator phase. When the stator and rotor poles are aligned, the self-inductance of the stator phase will be maximum and when the poles are unaligned, the self-inductance of the phase will be minimum. The phase inductance in a LSRM is a periodic function of the rotor position. At any given rotor position, the phase inductance also varies with the instantaneous phase current because of magnetic saturation. Therefore, in the inductance-based model, the position dependency of the phase inductance is represented by a limited number of Fourier series terms and the nonlinear variation of the inductance with current is expressed by means of polynomial functions, [23-25-33-34]:

$$L(x,i) = \sum_{k=0}^{m} L_k(i)\cos kN_r x \tag{1}$$

With $i, x\, et\, m$ are respectively the phase current, the position of the mover and the number of terms in the Fourier series.

The accuracy and stability of numerical simulations are the main challenges which should be met. To simplify the expression (1) only the first three terms of the Fourier series are considered. This leads to simplify the expression of the inductance given by the equation (2), [4-20]:

$$\begin{aligned} L(x,i_j) = {} & L_0(i_j) + L_1(i_j)\cos(N_r(x-(j-1)\frac{2\pi}{NN_r})) \\ & + L_2(i_j)\cos(2N_r(x-(j-1)\frac{2\pi}{NN_r})) \end{aligned} \tag{2}$$

With $L(x,i_j)$ *and* N are respectively the inductance associate to the phase j in the position x of the mover for the current i_j and the number of phase.

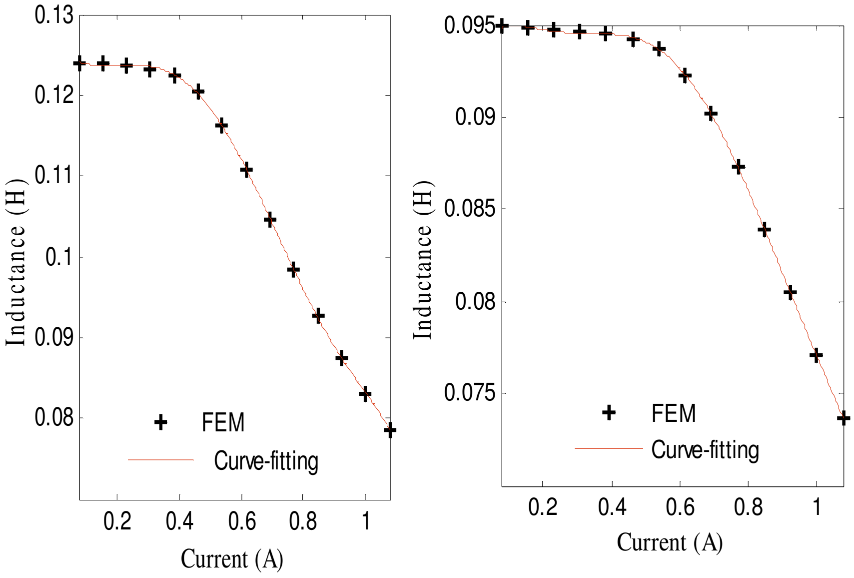
To determine the three coefficients $L_0, L_1\, et\, L_2$, we use the inductance at three positions: aligned position $L_c(i_j)$, unaligned position $L_{op}(i_j)$ and midway position between the above two positions $L_i(i_j)$. Note that $L_{op}(i_j)$ can be treated as a constant but, $L_c(i_j)$ and $L_i(i_j)$ are functions of the phase current i_j and can be approximated by the polynomials, [15-21-22-23-38]:

$$L_c(i_j) = \sum_{n=0}^{p} a_n i^n{}_j \tag{3}$$

$$L_i(i_j) = \sum_{n=0}^{p} b_n i^n{}_j \tag{4}$$

Where p is the order of the polynomials and a_n, b_n are the coefficients. In our research, $p = 6$ is chosen after we compare the fitting results of different p values, ($p = 3, p = 4, p = 5$ and $p = 6$ have been tried and compared). Result the inductance of aligned position $L_c(i_j)$ and midway position $L_i(i_j)$ are approximated respectively by the equation (5) and (6):

Figure 3 shows the evolution of the inductance versus current in the aligned and the midway postions.



a) b)

Fig. 3. Evolution of the inductance vs current in aligned (a) and midway positions (b)

$$L_c(i) = a_1 i^6 + a_2 i^5 + a_3 i^4 + a_4 i^3 + a_5 i^2 + a_6 i + a_7$$
$$\begin{array}{ll} \mathrm{a}_1 = -0.4883 & a_2 = 1.356 \\ \mathrm{a}_3 = -1.153 & a_4 = 0.1993 \\ \mathrm{a}_5 = 0.06603 & a_6 = -0.02222 \\ \mathrm{a}_7 = 0.1253 & \end{array} \tag{5}$$

$$L_i(i) = b_1 i^6 + b_2 i^5 + b_3 i^4 + b_4 i^3 + b_5 i^2 + b_6 i + b_7$$
$$\begin{array}{ll} b_1 = -0.3227 & b_2 = 1.186 \\ b_3 = -1.609 & b_4 = 0.9716 \\ b_5 = -0.2766 & b_6 = 0.03345 \\ b_7 = 0.09355 & \end{array} \tag{6}$$

Consequently, the three coefficients for the Fourier series can be computed as:

$$L_0 = \frac{1}{2}\left[\frac{1}{2}\left(L_c + L_{op}\right) + L_i\right] \tag{7}$$

$$L_1 = \frac{1}{2}\left(L_c - L_{op}\right) \tag{8}$$

$$L_2 = \frac{1}{2}\left[\frac{1}{2}\left(L_c + L_{op}\right) - L_i\right] \tag{9}$$

The stator phase inductance at the aligned position varies considerably with the stator phase current because of the magnetic saturation. The unaligned inductance does not vary much mainly because of the large reluctance that characterizes huge air gap in the flux path. It can be observed that the inductance characteristics versus current with three positions obtained by the proposed model closely match those obtained by finite element methods, figure 4.

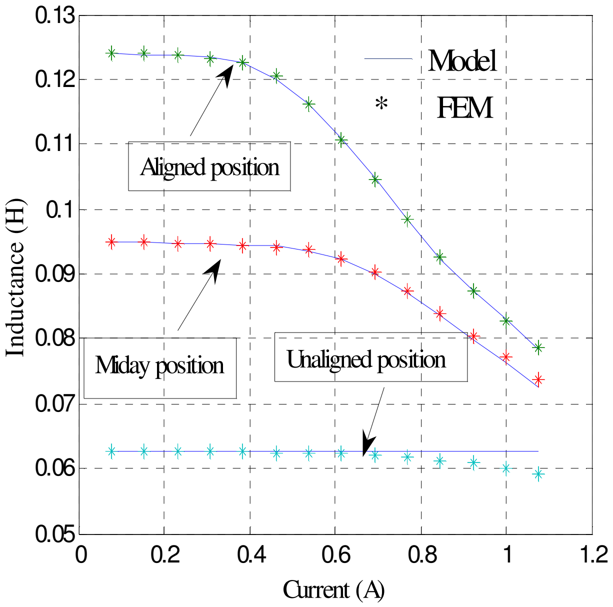


Fig. 4. Extreme left phase: Comparison of inductance versus current with three positions__Model, *FEM.

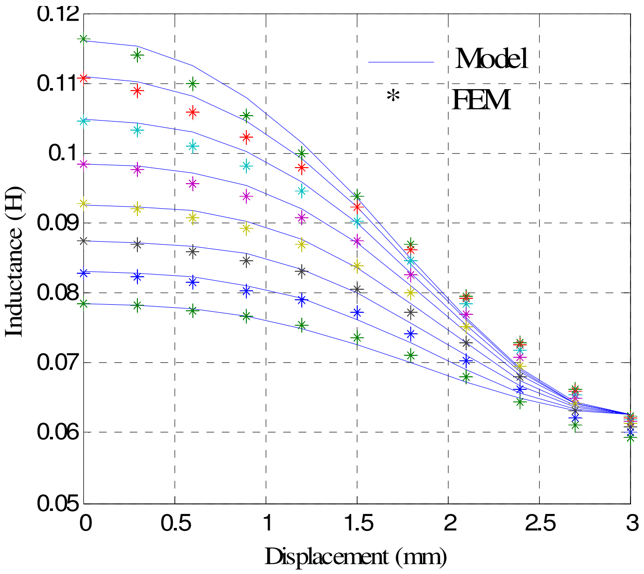


Fig.5. Extreme left phase: Comparison of inductance versus position with different currents__Model, *FEM.

A similar comparison is made for the characteristic of inductances versus positions for different values of currents, to show the effectiveness of the proposed model. Figure 5 shows a good agreement between the model and the finite element method MEF.

Multiplying the expression of inductance by the current (i), it gives the expression of linkage flux:

$$\varphi(i,x) = iL(i,x) \tag{10}$$

Figure 6 gives the comparison of linkage flux produced by the left extreme phase versus current for different positions. It can be observed that the linkage flux versus current for different position characteristics which are obtained by the proposed model closely match those obtained by finite element methods. These results prove the effectiveness of the proposed model.

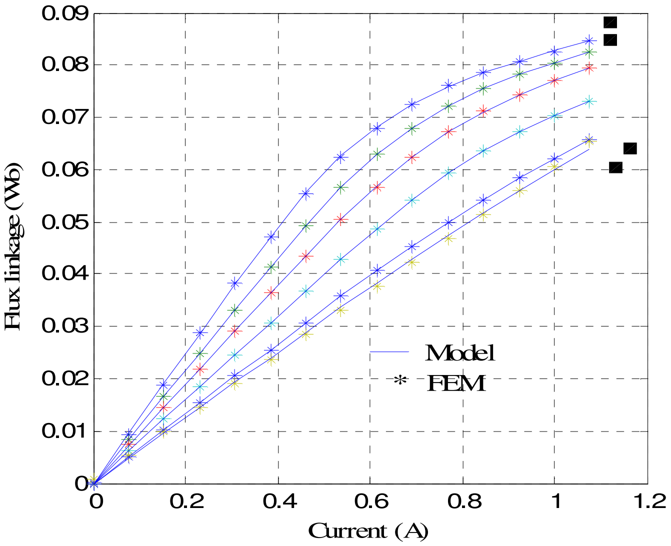


Fig. 6. Extreme left phase: Comparison of linkage flux versus current with different positions __Model, *FEM.

The total electromagnetic force is given by the following expression, [30-31-32-36-37]:

$$F = \sum_{j=1}^{N} F_j\left(i, x\right) \tag{11}$$

Where N is the number of phase, F_j the force of phase j and i_j the phase current. Consequently, the force F_j can be described by the following equation:

$$F_j\left(i, x\right) = \frac{\partial W_{c,j}}{\partial x} = \frac{\partial\left(\int_0^i L\left(x, i_j\right) i_j di_j\right)}{\partial x} \tag{12}$$

$L(x, i_j)$ is the inductance associate to the phase j in the position x of mover for the current i_j.

If the current is applied to a given phase, with stator and rotor teeth unaligned, the rotor will be attracted toward the balance position where the flux is maximum (aligned position) developing a force generally expressed by:

$$F_j(i, x) = \frac{1}{2}\frac{\partial L_j(x)}{\partial x} i_j^{\,2}\Bigg|_{[i]=cte} \tag{13}$$

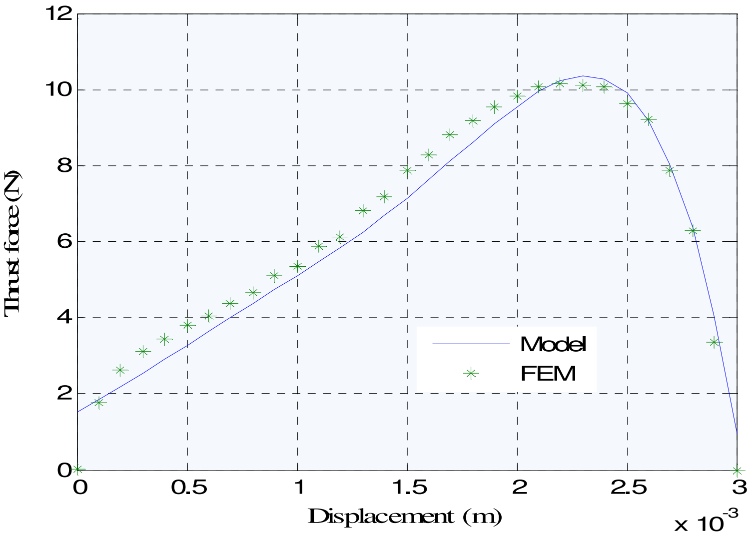
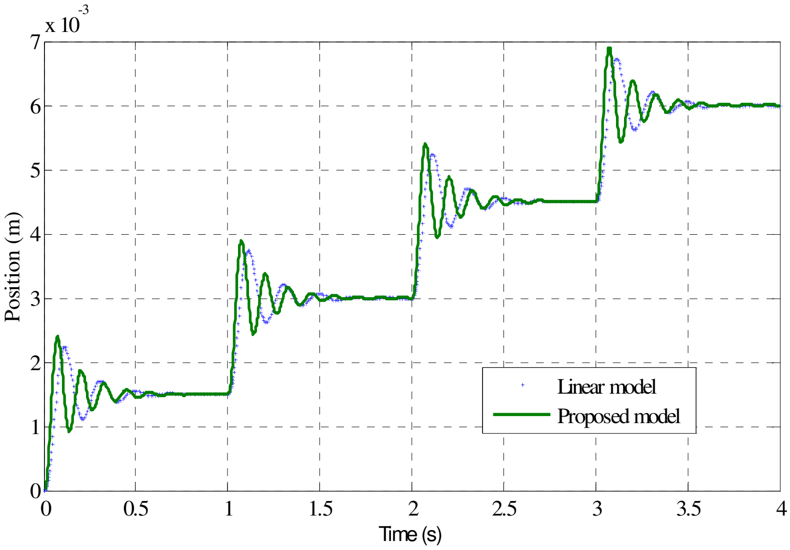
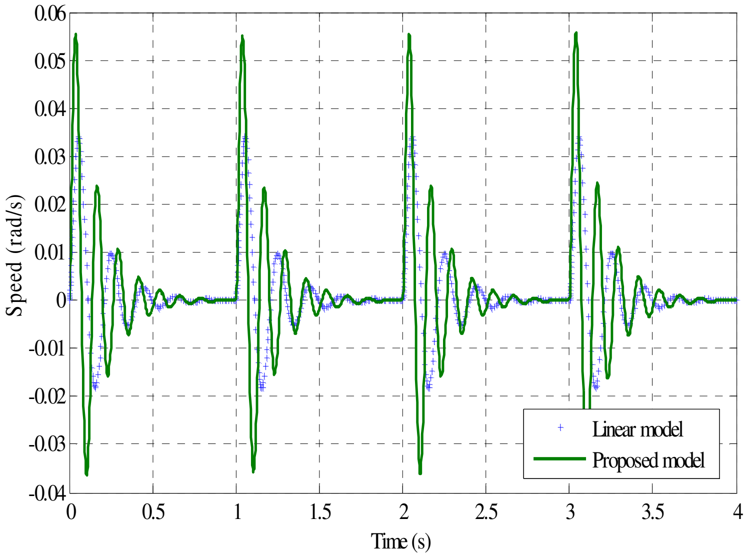


Fig. 7. Extreme left phase: Comparison of the thrust force as function of mover position __Model, *FEM.

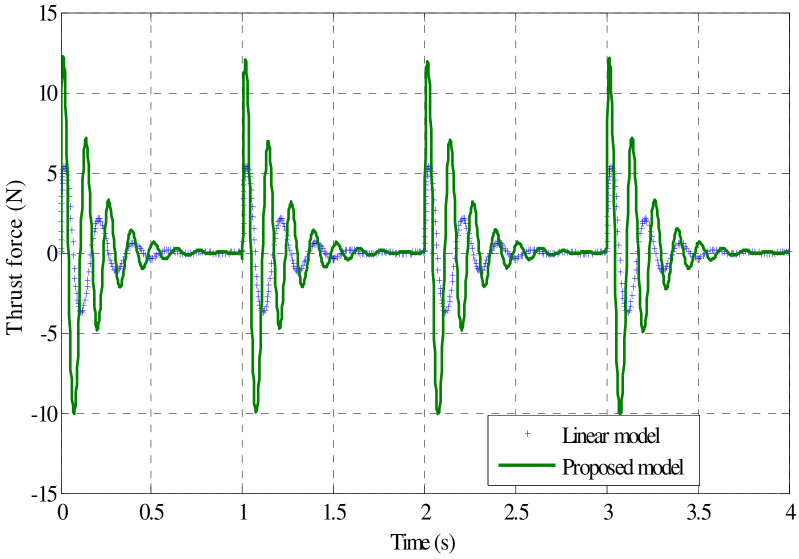
Figure 7 shows a perfect coincidence between the force characteristic determined by the proposed model and that taken via the finite element method (MEF). This represents a proof of the effectiveness of the proposed model.



a)



b)



c)

Fig. 8. Evolution of the position, speed and thrust force on four steps.

Figure 8, respectively illustrates the dynamic behavior of the machine for the model without saturation and with saturation (refined model) starting from the speed and of the angular position. The superposition of these results shows that the dynamics of the evolution of the position is respected. A static error due to friction affects the positions of balance: [5-41].

The refined model of the LSRM is characterized by a strongly oscillatory translation compared to the linear model. These oscillations are disturb the precision of the position and the constancy speed often required by many industrial applications and especially in the medical fields. This problem often leads to losses of synchronism, [11-17-18-26-28]

3. Conclusion

It is essential to have an accurate model of a Linear Switched Reluctance Motor that describes its static characteristics. It has been shown in this paper that there are different ways of modelling static characteristics of an LSRM. Developed analytical models consider the variation of the phase inductance with rotor position accounting for magnetic saturation. Results are compared to those obtained via the 2D-FEM. The comparison shows a reasonable agreement, proving the validity of the proposed approaches.

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